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Null deformed domain wall

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Abstract: We study null 1/4 BPS deformations of flat domain wall solutions (NDDW) in $\mathcal{N} = 2$, $d = 5$ gauged supergravity with hypermultiplets and vector multiplets coupled. These are uncharged *time-dependent* configurations and contain as special case, 1/2 supersymmetric flat domain walls (DW), as well as 1/2 BPS null solutions of the ungauged supergravity. Combining our analysis with the classification method initiated by *Gauntlett et al.*, we prove that *all the possible deformations of the DW have origin in the hypermultiplet sector* or/and *are null*. Here, we classify all the null deformations: we show that they naturally organize themselves into "gauging" (v-deformation) and "non gauging" (u-deformation). They have different properties: only in presence of v-deformation is the solution supported by a *time-dependent* scalar potential. Furthermore we show that the number of possible deformations *equals* the number of matter multiplets coupled. We discuss the general procedure for constructing explicit solutions, stressing the crucial role taken by the integrability conditions of the scalars as spacetime functions. Two analytical solutions are presented. Finally, we comment on the holographic applications of the NDDW, in relation to the recently proposed *time-dependent AdS/CFT*.

Keywords: Superstring Vacua, Supergravity Models, Gauge-gravity correspondence.

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1. Introduction

Studying time-dependent solutions in (Super)Gravity and String theory is an interesting and difficult task. Indeed our capacity of producing efficient cosmological models and generally describing our world relies on our control over time evolution. String theory, as a consistent theory of quantum gravity, should be able to provide a satisfactory answer to this and other outstanding related problems, such as the resolution of spacetime singularities. Unfortunately, it is very hard to keep the stability of such solutions under control, especially against quantum corrections. One of the crucial points is that a generic time-dependent solution is *not supersymmetric*, thus does not enjoy non renormalization properties associated to BPS configurations. Up to now the use of this property is the main way we have to study non perturbative phenomena.

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In this work we take the modest approach of considering an interesting class of time-dependent BPS configurations in $\mathcal{N}=2$ $d=5$ gauged supergravity with matter couplings. In doing so, we are in part inspired by [1 − 3],[1] where null deformations of $AdS_5 \times S^5$ are considered. These authors propose an extension of the AdS/CFT correspondence to such background, that is the near horizon limit of a null deformed stack of D3-branes (the null deformation of intersecting brane configurations has been recently considered in [5]). Such an extension is appealing because may allow one to inspect toy spacetime cosmological singularities via holography. In [1 − 3] it is argued that the dual theory corresponds to $\mathcal{N}=4$ super Yang-Mills theory (SYM) with time-dependent sources turned on. This picture has been supported and further investigated in [6]. An interesting property of the background analyzed in [1 − 3] is that the dilaton and, consequently, the gauge coupling of the dual theory are time-dependent (through a lightcone coordinate). Furthermore, as only the AdS_5 part is affected by the deformation, such solutions can be studied in full generality in the effective 5d (gauged) supergravity.

In our paper we investigate configurations of the form

$$\mathrm{d}s^2 = \beta^2(x^+, r)\left(-2k^2(x^+)\mathrm{d}x^+\mathrm{d}x^- + H(x^+, x^-, x^i, r)(\mathrm{d}x^+)^2 + (\mathrm{d}x^i)^2 + \mathrm{d}r^2\right).$$

We show that such configurations preserve 1/4-supersymmetry and include the null deformed AdS_5 space of [1 − 3] as special 1/2 BPS subcases. However, the above metric describes also another interesting 1/2-supersymmetric subclass - it contains flat domain wall solutions. This class of solutions has received a lot of attention mainly due to the role in the AdS/CFT correspondence, [4]. As solutions of gauged supergravity these are conjectured to be dual to the Renormalization Group (RG) flows of field theory couplings [7 − 13]. Domain walls are also a key ingredient of Brane world constructions [14 − 19]. More recently, in four dimensions, these solitons have been used as a laboratory for understanding mirror symmetry in flux/generalized geometry compactifications [20 − 22] and to explore transitions between the different cosmological vacua of the Landscape [23].

It is desirable to "combine" the two deformations of AdS we consider and verify whether and/or how the gauge/gravity correspondence applies to the resulting background. For these reasons, the study of "generalized" domain wall solutions remains an interesting area of study. Very recently non-supersymmetric charged domain walls have been investigated in [24] while BPS *gyratons* have been discussed in [25]. In both cases such configurations have been studied in the presence of vector multiplets coupling only.

In this work we shall consider all the matter couplings that are relevant for constructing domain walls. As shown in [26] the inclusion of hypermultiplets is crucial to have BPS domain walls interpolating between two AdS vacua and consequently to embed the domain wall solution of [27] (FGPW) in the $\mathcal{N}=2$ gauged supergravity, as holographic dual to an RG flow from an $\mathcal{N}=4$ to an $\mathcal{N}=1$ SYM. In [28] it has been shown that curved domain walls can be obtained only with hypermultiplets coupled. Currently there is a renewed interest in having a more systematical understanding of BPS solutions with hypermultiplets.

[1] In [3] null deformations of the near horizon limit of the general Dp-branes are considered and their holographic properties are studied. For $p \neq 3$, they are the null generalization of the $(p+2)$-dimensional domain walls of [4].

The full classification in $\mathcal{N}=2$ ungauged supergravity has been achieved in four and five dimensions in [29, 30]. Some steps towards this goal in the more complicated gauged case had been previously performed in five dimensions [31 – 33].

The configurations we present here are the first example of BPS time-dependent solutions in gauged supergravity with hypermultiplet coupled.

As an additional motivation, we would like to mention that the configurations we consider may be seen as the closest supersymmetry-preserving analogue of time-dependent solutions of [34, 35] describing Brane collision.

The organization of the paper is as follows. In order to fix the notation and be self-contained we present in section 2 the basic ingredients of the supergravity theory we are dealing with and we describe the main feature of (flat) domain wall solutions in $\mathcal{N}=2$ $d=5$ gauged supergravity.

Section 3 constitutes the main part of this paper and is devoted to the derivation and discussion of the BPS equations related to the metric above. Such an analysis is made in comparison with the original domain wall case which, using a non orthodox English terminology, we will refer to in the text as the "undeformed" configuration. We shall illustrate how the class of solitons under consideration admits a dual interpretation as null deformation of domain walls or deformation of a plane wave due to the "gauging". Taking the first point of view, we show that the null deformation naturally organizes itself into the contribution coming from the gauging, and another associated to the null solutions in the ungauged supergravity.

In section 4 the analysis of section 3 is given concrete applications and two explicit examples are constructed.

We finally collect our conclusions and propose possible developments in section 5.

All details of calculation that have not been given in the main text are presented in the appendix A and B. In appendix C we describe the parametrization of the coset space that appears in section 4.1. In appendix D we argue how "adapted coordinates" can be used to derive some insights into the possible solutions.

2. Domain wall in $\mathcal{N}=2$ $d=5$ gauged supergravity

This section is devoted mainly to review known facts on domain wall solution. Furthermore we remind here the basic ingredient of the supergravity theory we use, giving the formulae we use in our calculation.

2.1 Five-dimensional, $\mathcal{N}=2$ gauged supergravity

We start by recalling some of the most important features of five-dimensional, $\mathcal{N}=2$ gauged supergravity theories. Further technical details can be found in the original references [36 – 40].

The matter multiplets that can be coupled to $5D$, $\mathcal{N}=2$ supergravity are vector, tensor and hypermultiplets: the scalar φ of theory could a priori sit in any of these (or even be a combination of different types of scalars).

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The $(n_V + n_T)$ scalar fields of n_V vector and n_T tensor multiplets parameterize a "very special" real manifold $\mathcal{M}_{\rm VS}$, i.e., an $(n_V + n_T)$-dimensional hypersurface of an auxiliary $(n_V + n_T + 1)$-dimensional space spanned by coordinates $h^{\tilde I}$ $(\tilde I = 0, 1, \ldots, n_V + n_T + 1)$:

$$\mathcal{M}_{\rm VS} = \{h^{\tilde I} \in \mathbb{R}^{(n_V+n_T+1)} : C_{\tilde I\tilde J\tilde K} h^{\tilde I} h^{\tilde J} h^{\tilde K} = 1\}, \tag{2.1}$$

where the constants $C_{\tilde I\tilde J\tilde K}$ appear in a Chern-Simons-type coupling of the Lagrangian. The embedding coordinates $h^{\tilde I}$ have a natural splitting,

$$h^{\tilde I} = (h^I, h^M), \qquad (I = 0, 1, \ldots, n_V), \qquad (M = 1, \ldots, n_T), \tag{2.2}$$

where the h^I are related to the sub-geometry of the n_V vector multiplets, and the h^M refer to the n_T tensor multiplets. On $\mathcal{M}_{\rm VS}$, the $h^{\tilde I}$ become functions of the physical scalar fields, ϕ^x $(x = 1, \ldots, n_V + n_T)$. The metric on the very special manifold is determined via the equations

$$\begin{gathered} g_{xy} = h^{\tilde I}_x h_{y\tilde I}, \quad h^{\tilde I}_x \equiv -\sqrt{\tfrac{3}{2}}\,\partial_x h^{\tilde I}, \quad h_{\tilde I} \equiv C_{\tilde I\tilde J\tilde K} h^{\tilde J} h^{\tilde K}, \quad h_{\tilde I x} \equiv \sqrt{\tfrac{3}{2}}\,\partial_x h_{\tilde I}, \\ h^{\tilde I} h_{\tilde J} + h^{\tilde I}_x\, g^{xy}\, h_{y\tilde J} = \delta^{\tilde I}_{\tilde J}, \qquad h^{\tilde I} h_{\tilde I} = 1, \qquad h^{\tilde I} h_{\tilde I x} = 0. \end{gathered} \tag{2.3}$$

The scalars q^X $(X = 1, \ldots 4n_H)$ of n_H hypermultiplets, on the other hand, take their values in a quaternionic-Kähler manifold $\mathcal{M}_{\rm Q}$ [41], i.e., a manifold of real dimension $4n_H$ with holonomy group contained in $SU(2) \times USp(2n_H)$. We denote the vielbein on this manifold by f^{iA}_X, where $i = 1, 2$ and $A = 1, \ldots, 2n_H$ refer to an adapted $SU(2) \times USp(2n_H)$ decomposition of the tangent space. The hypercomplex structure is (-2) times the curvature of the $SU(2)$ part of the holonomy group,[2] denoted as $\mathcal{R}^{rZX}$ $(r = 1, 2, 3)$, so that the quaternionic identity reads

$$\mathcal{R}^r_{XY}\mathcal{R}^{sYZ} = -\tfrac{1}{4}\,\delta^{rs}\,\delta_X{}^Z - \tfrac{1}{2}\,\varepsilon^{rst}\,\mathcal{R}^t_X{}^Z. \tag{2.4}$$

Besides these scalar fields, the bosonic sector of the matter multiplets also contains n_T tensor fields $B^M_{\mu\nu}$ $(M = 1, \ldots, n_T)$ from the n_T tensor multiplets and n_V vector fields from the n_V vector multiplets. Including the graviphoton, we thus have a total of $(n_V + 1)$ vector fields, A^I_μ $(I = 0, 1, \ldots, n_V)$, which can be used to gauge up to $(n_V + 1)$ isometries of the quaternionic-Kähler manifold $\mathcal{M}_{\rm Q}$ (provided such isometries exist). These symmetries act on the vector-tensor multiplets by a representation $t_{I\tilde J}{}^{\tilde K}$, where in the pure vector multiplet sector $t_{IJ}{}^K = f_{IJ}{}^K$ are the structure constants, and the other components also satisfy some restrictions [38, 42, 40]. The transformations should leave the defining condition in (2.1) invariant, hence

$$t_{I(\tilde J}{}^{\tilde M} C_{\tilde K\tilde L)\tilde M} = 0. \tag{2.5}$$

The very special Kähler target space then has Killing vectors

$$K^x_I(\phi) = -\sqrt{\tfrac{3}{2}} t_{I\tilde J}{}^{\tilde K} h^x_{\tilde K} h^{\tilde J}. \tag{2.6}$$

[2] In fact, the proportionality factor includes the Planck mass and the metric, which are implicit here.

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There may be more Killing vectors, but these are the ones that are gauged using the gauge vectors in the vector multiplets.

The quaternionic Killing vectors $K_I^X(q)$ that generate the isometries on $\mathcal{M}_\mathrm{Q}$ can be expressed in terms of the derivatives of $SU(2)$ triplets of Killing prepotentials $P_I^r(q)$ ($r = 1,2,3$) via

$$D_X P_I^r = \mathcal{R}^r_{XY} K_I^Y, \qquad \Leftrightarrow \qquad \begin{cases} K_I^Y = -\frac{4}{3}\mathcal{R}^{rYX} D_X P_I^r \\ D_X P_I^r = -\varepsilon^{rst}\mathcal{R}^s_{XY} D^Y P_I^t, \end{cases} \tag{2.7}$$

where D_X denotes the $SU(2)$ covariant derivative, which contains an $SU(2)$ connection ω_X^r with curvature $\mathcal{R}^r_{XY}$:

$$D_X P^r = \partial_X P^r + 2\,\varepsilon^{rst}\omega_X^s P^t, \qquad \mathcal{R}^r_{XY} = 2\,\partial_{[X}\omega^r_{Y]} + 2\,\varepsilon^{rst}\omega^s_X\omega^t_Y. \tag{2.8}$$

The prepotentials satisfy the constraint

$$\frac{1}{2}\mathcal{R}^r_{XY}K_I^X K_J^Y - \varepsilon^{rst}P_I^s P_J^t + \frac{1}{2}f_{IJ}{}^K P_K^r = 0, \tag{2.9}$$

where $f_{IJ}{}^K$ are the structure constants of the gauge group.

In the following, we will frequently switch between the above vector notation for $SU(2)$-valued quantities such as P_I^r, and the usual (2×2) matrix notation,

$$P_{Ii}{}^j \equiv \mathrm{i}\,\sigma_{ri}{}^j P_I^r. \tag{2.10}$$

An important difference in geometrical significance between the very special Killing vectors $K_I^x(\phi)$ in (2.6) and the quaternionic ones $K_I^X(q)$ in (2.7), is that the former do not arise as derivatives of Killing prepotentials, because there is no natural symplectic structure on the real manifold $\mathcal{M}_\mathrm{VS}$ that could define a moment map.[3]

Turning on only the metric and the scalars, the general Lagrangian of such a gauged supergravity theory is

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y - \frac{1}{2}g_{XY}\partial_\mu q^X\partial^\mu q^Y - g^2\mathcal{V}(\phi,q), \tag{2.11}$$

whereas the supersymmetry transformation laws of the fermions are given by

$$\delta\psi_{\mu i} = \nabla_\mu\epsilon_i - \omega_{\mu i}{}^j\epsilon_j - \frac{\mathrm{i}}{\sqrt{6}}\,g\,\gamma_\mu P_i{}^j\epsilon_j, \tag{2.12}$$

$$\delta\lambda_i^x = -\frac{\mathrm{i}}{2}\gamma^\mu(\partial_\mu\phi^x)\epsilon_i - g\,P_i{}^{jx}\epsilon_j + g\,\mathcal{T}^x\epsilon_i, \tag{2.13}$$

$$\delta\zeta^A = \frac{\mathrm{i}}{2}f_X^{iA}\gamma^\mu(\partial_\mu q^X)\epsilon_i - g\,\mathcal{N}^{iA}\epsilon_i. \tag{2.14}$$

[3]The moment maps are related to the fact that the isometries should preserve complex structures. Therefore, they are absent in the real manifold. In 4 dimensions, the scalar manifold of the vector multiplets does have a complex structure. Hence, in that case this sector would also have a moment map structure [43]. This suggests that in four dimensions the same comparison may go along different lines.

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Here, ψ^i_μ, λ^x_i, ζ^A are the gravitini, gaugini (tensorini) and hyperini, respectively, g denotes the gauge coupling, the $SU(2)$ connection $\boldsymbol{\omega}_\mu$ is defined as $\omega_{\mu i}{}^j = (\partial_\mu q^X)\omega_{Xi}{}^j$, and

$$
\begin{aligned}
P^r &= h^I(\phi) P^r_I(q), && (2.15)\\
P^r_x &= -\sqrt{\frac{3}{2}}\partial_x P^r = h^I_x P^r_I, \qquad P^{rx} = g^{xy} P^r_y, && (2.16)\\
\mathcal{N}^{iA} &= \frac{\sqrt{6}}{4} f^{iA}_X(q) h^I(\phi) K^X_I(q), && (2.17)\\
\mathcal{T}^x &= \frac{\sqrt{6}}{4} h^I(\phi) K^x_I(\phi). && (2.18)
\end{aligned}
$$

As a general fact in supergravity, the potential is given by the sum of "squares of the fermionic shifts" (the scalar expressions in the above transformations of the fermions):

$$
\mathcal{V} = -4P^r P^r + 2P^r_x P^r_y g^{xy} + 2\mathcal{N}^{iA}\mathcal{N}^{jB}\varepsilon_{ij} C_{AB} + 2\mathcal{T}^x \mathcal{T}^y g_{xy}, \tag{2.19}
$$

where C_{AB} is the (antisymmetric) symplectic metric of $USp(2n_H)$.

Using the explicit form of the Killing vector, (2.6), in (2.18), one finds that this expression vanishes if the transformation matrix t involves only vector multiplets. This is clear because then $t_{IJ}{}^K = f_{IJ}{}^K$, hence antisymmetric. Therefore, the shift $\mathcal{T}^x$ in the above expressions is non-vanishing only if there are charged tensor multiplets in the theory.[4] Since $\mathcal{T}^x$ appears in (2.13) with the unit matrix in $su(2)$ space, it must vanish on a BPS-domain wall solution for compatibility with the spinor projector (see [26, footnote 8] and [44]). Furthermore, unlike the shifts P^r_x and $\mathcal{N}^{iA}$, $\mathcal{T}^x$ is a purely "D-type" term, in the sense that it is completely unrelated to derivatives of the moment map P^r. Thus, for BPS-domain walls in $5D$, $\mathcal{N}=2$ supergravity (and in fake supergravity as well [28]), non-trivial tensor multiplets can not play an important rôle, and we can limit our remaining discussion to the case $n_T = 0$, i.e., to supergravity coupled to vector and/or hypermultiplets only. This also means that the index $\tilde{I}$ simply becomes the index I in all previous equations, and the index M disappears.

Before reviewing the BPS domain wall solutions, let us present the integrability conditions of the Gravitini variation (2.12). Following [32], all the information contained in (2.12) for uncharged BPS configurations in presence of matter, can be cast in the compact form:

$$
\left(\frac{1}{4}\Omega_{cd}{}^{ab}\gamma_{ab}\delta_i{}^j - iR^r_{cd}(\sigma_r)_i{}^j - \frac{2g}{\sqrt{6}}\gamma_{[c}D_{d]}P^r(\sigma_r)_i{}^j + \frac{g^2}{2}W^2\gamma_{cd}\right)\epsilon_j = 0, \tag{2.20}
$$

where $D_\mu P^r \equiv \partial_\mu \varphi^\Lambda D_\Lambda P^r$ and $R^r_{\mu\nu} \equiv \partial_\mu q^X \partial_\nu q^Y R^r_{XY}$ are the pull-back of the SU(2)-covariant derivative of the moment map and of the SU(2)-curvature, respectively. W is the *superpotential*, $P^r P^r \equiv \frac{3}{2}W^2$ (the normalization is chosen for convenience). Imposing (2.20) together with the BPS conditions of the matter field, is sufficient to ensure the Einstein equation for the metric, for time-like BPS configurations [32] (i.e. when the vector bilinear constructed by the covariantly constant spinor $V^\mu \equiv 1/2\bar{\epsilon}^i\gamma^\mu\epsilon_i$ is time-like),

[4] In five dimensions, tensor multiplets that are not charged under some gauge group are equivalent to vector multiplets. We always assume that all uncharged tensor multiplets are converted to vector multiplets.

or more precisely, when there is no light-like projector on the covariantly constant spinor ($\gamma_{\mp}\epsilon = 0$). We will see in section 3, how the equations of motion impose extra-condition over the metric in the light-like case ($V^{\mu}V_{\mu} = 0$).[5]

2.2 BPS-domain walls in supergravity

Now we will remind to the reader of some known facts about domain wall configurations, pointing out some novel features along the way. This subject has been extensively studied in the literature, mainly as an application/extention of the AdS/CFT correspondence and as phenomenological model with large extra-dimensions (Brane world). The relevance of such configuration justified the derivation of an “effective” supergravity approach [12] known as *Fake supergravity* [45], valid for any space-time dimensions. The explicit relation of this powerful tool for constructing domain wall solutions, with the full-fledged $\mathcal{N} = 2$ $D = 5$ gauged supergravity was first uncovered in [28], and further explored in [46].[6] Remarkably, the same first order formalism (extended to include dS-brane in [48, 49]) applies also to Friedmann-Robertson-Walker cosmology [50], motivating the derivation of the domain wall/Cosmology correspondence [46, 51].

We will review the subject from a different prospective to usual (cfr. [28]). The normal procedure is to start with a domain wall ansatz for the metric,

$$\mathrm{d}s^2 = \mathrm{e}^{2U(r)} g_{\bar{\mu}\bar{\nu}}(x)\, \mathrm{d}x^{\bar{\mu}}\, \mathrm{d}x^{\bar{\nu}} + \mathrm{d}r^2, \tag{2.21}$$

and assume that the scalar fields depend only on the fifth dimension r (we indicate with a bar the indices running over the remaining four dimensions). By definition of a domain wall, the four dimensional metric $g_{\bar{\mu}\bar{\nu}}$ of the wall has constant curvature that BPS equations fixed to be non positive. When this is negative (AdS_4) the domain wall is said to be *curve* or AdS-sliced, while is called *flat* or Minkowski-sliced in case of zero curvature.

We shall instead begin by requiring that the scalar fields depending only on one space-time *spatial* coordinate, that for convenience we take as fifth coordinate. This is equivalent to assume that the metric is a *warped product* of a radial coordinate times a generic four dimensional metric. So any a priori assumption is made about the form of $g_{\bar{\mu}\bar{\nu}}$ in (2.21),[7] a part the fact that it does not dependent on r. We will show of this weaker requirement is sufficient to identify a domain wall solution. Following the analysis of [32], further extended in [52], we decompose the derivative of the quaternionic scalars as:

$$\partial_5 q^X = MK^X + 2v_r D^X P^r. \tag{2.22}$$

As a consequence, the hyperini equation (2.14) reduces to

$$\left[\sqrt{\frac{3}{2}} ig\delta_i{}^j + \gamma_5 M\delta_i{}^j - iv^r\gamma_5(\sigma_r)_i{}^j\right]\epsilon_j = 0. \tag{2.23}$$

[5] Actually, a subtlety that has never been put in evidence is that a domain wall solution is always light-like, although in a trivial way. This point will be clarified in section 3.1.

[6] The relation between Fake supergravity and $\mathcal{N} = 4$, $d = 5$ gauged supergravity has been studied in [47].

[7] Our study at this stage can not exclude the existence of different supersymmetric solutions than the domain walls, where the scalars depend only on one coordinate that does *not factorize* in the metric. Such possibility could be interesting in the contest of holography.

Now, the other crucial physical requirement of the solution enters the game, i.e. it must be *uncharged*. Under this condition, the equations of motion for the gauge field reduces to

$$K^X \partial_a q^Y g_{XY} = 0, \tag{2.24}$$

which immediately gives $M = 0$. Thus (2.23) becomes

$$\gamma_5 \epsilon_i = \alpha^r (\sigma_r)_i{}^j \epsilon_j, \tag{2.25}$$

where the phase α^r ($\alpha^r \alpha_r = 1$) is given by $\alpha^r \equiv \sqrt{\frac{2}{3}}\frac{1}{g} v^r$.

The analysis of the gaugini equation (2.13) yields to an analogous result. By imposing $\partial_a \phi^x = \delta_a^5 \partial_5 \phi^x$, one gets

$$\left(\partial_5 \phi^x \gamma_5 \delta_i{}^j + 2g P^{xr} (\sigma_r)_i{}^j\right) \epsilon_j = 0. \tag{2.26}$$

The above equation is easily seen to be equivalent to (2.25) plus

$$\partial^x P^r = \frac{\alpha^r \partial_5 \phi^x}{\sqrt{6} g}. \tag{2.27}$$

Hence the first order equations for the scalars of hypermultiplets and vector multiplets can be written in a unified framework as:

$$\partial_5 \varphi^\Lambda = 2\sqrt{\frac{3}{2}} g \alpha^r D^\Lambda P^r, \qquad \Lambda = 1, \ldots, n_V + 4n_H, \tag{2.28}$$

where

$$\varphi^\Lambda \equiv \begin{cases} q^X, & \Lambda = 1, \ldots, 4n_H = X \\ \phi^x, & \Lambda = 4n_H + 1, \ldots, 4n_H + n_V = x + 4n_H \end{cases},$$

$$D^\Lambda P^r \begin{cases} D^X P^r, & \Lambda = 1, \ldots, 4n_H = X \\ \partial^x P^r, & \Lambda = 4n_H + 1, \ldots, 4n_H + n_V = x + 4n_H \end{cases},$$

However, let us emphasize that the vector multiplet scalar sector is constrained by a stronger condition, due to (2.27), i.e. $\partial_x P^r // \alpha^r$.[8]

We remember that, up to now, we did not assume any guess for the metric g of the four dimensional slice orthogonal to r. Its form will be determined by the integrability conditions of the gravitini. Taking in account that, from (2.28) we find,

$$\begin{aligned} D_a P^r &= \partial_a \varphi^\Lambda D_\Lambda P^r = \\ &= 3\sqrt{\frac{3}{2}} g \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W\right) \delta_a^5 \alpha^r, \qquad \gamma \equiv -\alpha^s Q^s, \end{aligned} \tag{2.29}$$

equation (2.20) becomes

$$\left\{1/2 \Omega_{cd}{}^{ab} \gamma_{ab} - g^2 \left[3\left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W\right)\left(\delta_c^5 + \delta_d^5\right) - W^2\right] \gamma_{cd}\right\} \epsilon_i = 0, \tag{2.30}$$

[8]This property forces the domain wall supported by vector multiplets to be flat, as first observed in [28].

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where (2.25) is crucial to reduce the above expression to a combination of gamma matrices. Now, differently from the case we will discuss in the next section, no other projection condition can be enforced because we are looking for 1/2 BPS solution. Hence, (2.30) must be trivial and the curvature "diagonal", i.e.

$$\Omega_{cd}{}^{ab} = 2g^2 \left[3 \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) (\delta_c^5 + \delta_d^5) - W^2 \right] \delta_c^{[a} \delta_d^{b]}. \tag{2.31}$$

Using

$$\begin{aligned} \dot{W} \equiv \partial_r W &= \sqrt{\frac{2}{3}} \partial_r \varphi^\Lambda D_\Lambda P^s Q^s \\ &= -3g \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \gamma, \end{aligned} \tag{2.32}$$

(2.31) can be cast as

$$\Omega_{cd}{}^{ab} = 2g^2 \left[-\frac{\dot{W}}{g\gamma} (\delta_c^5 + \delta_d^5) - W^2 \right] \delta_c^{[a} \delta_d^{b]}. \tag{2.33}$$

The above expression is sufficient to show that the four dimensional slice is a space of non positive constant curvature. First we observe that for a warped metric of the form (2.21), the curvature can be written as

$$\Omega_{cd}{}^{\bar{a}\bar{b}} = \bar{\Omega}_{cd}^{\bar{a}\bar{b}} - 2(\dot{A})^2 \delta_c^{[\bar{a}} \delta_d^{\bar{b}]} \tag{2.34}$$

$$\Omega_{cd}{}^{\bar{a}5} = -2(\ddot{A} + (\dot{A})^2) \delta_c^{[\bar{a}} \delta_d^{5]} \tag{2.35}$$

where $\bar{\Omega}^{\bar{a}\bar{b}} = 1/2 e^{-2A} \bar{\Omega}_{\bar{c}\bar{d}}^{\bar{a}\bar{b}} e^{\bar{c}} \wedge e^{\bar{d}}$ is the intrinsic curvature associated to the metric g. The comparison between (2.34) and (2.33) implies that $\bar{\Omega}^{\bar{a}\bar{b}}$ is proportional to $e^{\bar{a}} \wedge e^{\bar{b}}$ via a function of r *only*, that can be reabsorbed in the warp-factor. In practice this means that A can be taken *such that*

$$\left(g^2 W^2 - (\dot{A})^2 \right) e^{2A} = \frac{1}{L^2}, \tag{2.36}$$

where $\bar{R} = -\frac{12}{L^2}$ the constant scalar curvature of g. It remains to demonstrate that $L \in \mathcal{R}$, i.e. is the length of AdS_4 (that reduce to Minkowski for $L = 0$). From the comparison between (2.35) and (2.33) it follows

$$\ddot{A} + (\dot{A})^2 = g^2 \left(\frac{\dot{W}}{g\gamma} + W^2 \right). \tag{2.37}$$

Using (2.36) we conclude that $\dot{A} = g\gamma W$, hence $(g^2 W^2 - (\dot{A})^2), L^2 \geq 0$, because $0 \leq \gamma^2 \leq 1$.

Let us summarize what we have presented in this section. It has been shown that the well known domain wall solutions are the unique BPS solutions that can be written as in (2.21) with the scalars depending only on r.

In other words we have displayed that assigning (2.28) is sufficient to get a domain wall. In this way we can establish a one-to-one correspondence between the projector (2.25) and domain wall solutions.

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In the next section we will study a supersymmetric deformation of these solutions. In order to do so we will focus on the flat domain walls (DW), i.e. $\gamma^2 = 1$. We conclude by observing, for future reference, that in this case the metric (2.21) can be conveniently expressed as a *conformally flat* metric

$$\mathrm{d}s^2 = \beta^2(x^5)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{2.38}$$

where x^5 is related to r by the change of coordinate $\mathrm{d}r = \beta(x^5)\mathrm{d}x^5$, with $\beta(x^5) = e^{A(r)}$. The BPS equations become

$$\frac{\dot\beta}{\beta^2} = g\gamma W, \tag{2.39}$$

$$\dot\varphi^\Lambda = -3g\beta\gamma\partial^\Lambda W, \tag{2.40}$$

the dot now indicating the derivative with respect to the new coordinate x^5.

3. Null deformation

Now we want to consider together with (2.25) the projector

$$\gamma_0\epsilon_i = \pm\gamma_1\epsilon_i. \tag{3.1}$$

As will be shown clearly below, the resulting configuration can be seen as the generalization of the light-like deformation of $AdS_5 \times S^5$ studied in [1–3], from an effective five dimensional point of view.[9] For convenience, we name it as "Null-deformed domain wall", or shortly NDDW, while we will refer to the non deformed flat domain wall simply as DW.

It is convenient to change our frame from the ordinary Minkowski to the lightcone one. We define $E^\pm \equiv \frac{E^0 \pm E^1}{\sqrt{2}}$, in order to have $\eta_{\pm\mp} = -1$. The (3.1) now reads $\gamma_\mp\epsilon_i = 0$.

It is easy to verify that the two conditions over the covariant spinor are consistent. This point will be discussed in section 3.1 from the prospective of the classification method [53]. We will argue that, to some extent, the NDDW is the *most general non static deformation* of the DW.

The introduction of (3.1) reduces the amount of supersymmetry from 1/2 to 1/4 and, as a consequence the DW metric (2.38) is deformed. In order to study such deformation, we will consider the following metric (see the appendix for more details)

$$\mathrm{d}s^2 = \beta^2(x^+, r)\left(-2k^2(x^+)\mathrm{d}x^+\mathrm{d}x^- + H(x^+, x^-, x^i, r)(\mathrm{d}x^+)^2 + \mathrm{d}r^2 + (\mathrm{d}x^i)^2\right). \tag{3.2}$$

The above metric represents the most general light-like deformation of (2.38), where the Minkowski slice has been replaced by a generic PP wave and the conformal factor β admits a dependence on lightcone coordinate x^+. It reduces to the one studied in [1] for β and r taken to be respectively the warp-factor and the radial coordinate of AdS_5 in the "Brinkman form" [1, eq.(5)] respectively.

[9] However, as for the non deformed DW, the uplifting to ten dimensions of our 5d model, is more involved than in the vacuum case, and is in general unknown.

In order to attack the problem, we follow the same strategy as in the previous section. By first we discuss the BPS equation for the scalars. Then, we use it to determine the curvature (to be compared with the one resulting from the ansatz) via the integrability condition (2.20).

First of all, we observe that, including both the gaugini and hyperini equations (2.13), (2.14) a term of the form $\gamma^a \partial_a \varphi^\Lambda$, the projector (2.25) allows the presence of a non zero $\partial_\pm \varphi^\Lambda$ component, which does not interfere with $\partial_5 \varphi^\Lambda$, remaining formally the same as for the DW. This means that equations (2.23) and (2.26) are untouched.

Similarly to (2.22), we decompose $\partial_\pm q^X$ in (again the e.o.m imposes $K^X \partial_\mu q^Y g_{XY} = 0$)

$$\partial_\pm q^X = v_s D^X P^s + u^X, \tag{3.3}$$

where u^X is orthogonal to K^X and $D^X P^s$. This decomposition is not only convenient for practical reasons, but also the two terms play different roles in the BPS equations (cfr. (3.6)). This reflects their different origin: while $v_s D^X P^s$ is associated to the gauging, u^X is related to the ungauged theory.

Taking into account that we want to study 1/4-BPS configurations, it must be $v^r // \alpha^r$. Indeed introducing another SU(2) direction is equivalent to add an extra projector condition like (2.25), as can be seen from the gravitini integrability condition (GIC) (2.20).

The analysis of the gaugini equations goes along the same lines.

We can write the kinetic term of the scalars as

$$\begin{aligned}
\partial_a q^X &= \left(2\sqrt{\frac{3}{2}} g v \alpha_s D^X P^s + u^X\right) \delta_a^\pm + 2\sqrt{\frac{3}{2}} g \alpha_s D^X P^s \delta_a^5, \\
\partial_a \phi^x &= \left(2\sqrt{\frac{3}{2}} g w \alpha_s \partial^x P^s + u^x\right) \delta_a^\pm + 2\sqrt{\frac{3}{2}} g \alpha_s \partial^x P^s \delta_a^5.
\end{aligned} \tag{3.4}$$

As in hypermultiplet case, the vector u^x is orthogonal to $\partial^x P^s$ (similar considerations hold), while the normalization of v and w is chosen for convenience to have $\partial_\pm q^X = v \partial_5 q^X + u^X$ and $\partial_\pm \phi^x = w \partial_5 \phi^x + u^x$ respectively.

At first sight, the "democratic" behavior of the scalars appearing in the DW case, eq. (2.28), (which is related to the success of the Fake supergravity approach) seems to be spoiled, because a priori v and w can be generic (unrelated) functions of the moduli space.

Following the same procedure as in the previous section, we can specialize the integrability condition (2.20) to the NDDW configuration, computing $D_a P^r$ and making use of (2.25). The exact expression is not so illuminating and is presented in the appendix, (A.12).

What is instead crucial, is that now the curvature Ω^{ab} (as well as the Ricci tensor) acquires "off-diagonal" terms (i.e. not proportional to $\delta_{[c}{}^a \delta_{d]}{}^b$) related to the deformation. Again this is a consequence of the new projection condition (3.1). The detail of this computation may be found in the appendix, equations (A.13)–(A.16).

Let us remark that the curvature is completely determined by the integrality condition up to the $\Omega_{\pm\tilde{b}}{}^{\mp\tilde{a}}$ component. This feature is common to all the BPS solutions associated to the projector (3.1). Indeed the component $\Omega_{\pm\tilde{b}}{}^{\mp\tilde{a}}$ always cancels out because it enters the integrability conditions multiplied by $\gamma_{\mp}$, that is zero on ϵ_i.[10]

Comparing the result we get from the GIC with the curvature computed starting by the ansatz (3.2), we obtain the BPS equations:

$$\frac{\dot{\beta}}{\beta^2} = g\gamma W, \qquad \gamma^2 \equiv (-\alpha^s Q^s)^2 = 1, \tag{3.5}$$

$$\frac{1}{\beta}(\frac{\dot{\beta}}{\beta^2})' = -3g^2\left(v\,\partial^X W\partial_X W + w\,\partial^x W\partial_x W\right), \tag{3.6}$$

$$\partial_i\partial_- H = \partial_-^2 H = \partial_-\dot{H} = 0. \tag{3.7}$$

In force of eq. (3.7) H may be decomposed as:

$$H(x^+, x^-, x^i, r) = \tilde{H}(x^+, x^i, r) + H_-(x^+)\,x^-. \tag{3.8}$$

Let us note that the relation between the derivative with respect to r and the superpotential, (3.5), stays the same as in the DW case. In addition, we find again that $\gamma^2 = 1$. This is not surprising, in fact as this is our input (as announced at the beginning, we restrict ourselves to null deformation of the flat domain wall metric (2.38)) rather than a requirement of supersymmetry. Indeed, generalizing the metric ansatz (3.2), it is possible to study curved domain wall deformation without changing the integrability conditions (A.13)–(A.16). An other interesting remark relates to the absence of u^Λ in the BPS equations (3.5)–(3.7). This is a first indication of the intrinsics difference between u and v, w-deformations.

However, the relation between u^Λ and the metric comes from the Einstein equation $((\pm\pm) \equiv (01)$ component, to be precise). As per usual, and as explained above, the first-order equations of light-like BPS solution [53, 32] are not sufficient to solve all the equations of motion and fix the ansatz completely. Explicitly we find

$$\begin{aligned} R_\pm{}^\mp &= -9g^2\left(v\,\partial^X W\partial_X W + w\,\partial^x W\partial_x W\right) - u^\Lambda u_\Lambda \\ &= \frac{3}{\beta}\left(D' - D\frac{2kk' + 1/2\partial_- H}{k^2} + \frac{1}{2}g\gamma W\dot{H}\right) + \frac{1}{2\beta^2}\left(\sum_i \partial_i^2 H + \ddot{H}\right), \end{aligned} \tag{3.9}$$

where in parallel with (3.5) we introduce $D \equiv \frac{\beta'}{\beta^2}$. This equation is crucial to relate the function H, characterizing the metric, to the scalars and the warp-factor, determining the solution.

This is the only extra requirement coming from the equations of motion, (apart from $K^X\partial_a q^Y g_{XY} = 0$, used since the beginning) that otherwise are identically satisfied. Indeed, it is easy to verify that the equations of motion for the scalars reduce to the one for the undeformed configuration, and, as in that case, are identically satisfied. This result is somewhat expected because the null contribution to the kinetic term is *traceless* thus does not enter in the laplacian (for the details of the calculation we refer the reader to the appendix B).

[10]This does not happen for the other component of $\Omega^{\mp a}$ because of the symmetry of the curvature. Furthermore we remind that, due to (3.1), $\gamma_{\pm\mp} \equiv 1/2[\gamma_\pm, \gamma_\mp]$ (that is not equal to $\gamma_\pm\gamma_\mp$) is not zero on ϵ but proportional to the identity.

The last non trivial constraint comes from the integrability conditions for the scalars (SIC). Taking a unifying notation

$$\partial_\mu \varphi^\Lambda = \beta \left[\left(-3g\gamma v_{(\Lambda)} \partial^\Lambda W + u^\Lambda \right) \delta_\mu^+ - 3g\gamma \partial^\Lambda W \delta_\mu^r \right], \tag{3.10}$$

where

$$v_{(\Lambda)} \equiv \begin{cases} v, & \Lambda = 1, \ldots, 4n_H = X \\ w, & \Lambda = 4n_H + 1, \ldots, 4n_H + n_V = x + 4n_H \end{cases},$$

the integrability condition $(\partial_+ \partial_r - \partial_r \partial_-)\varphi^\Lambda = 0$ implies

$$\begin{aligned} &\left(-3g^2 W v_{(\Lambda)} + 9g^2 \partial^\Sigma W \partial_\Sigma v_{(\Lambda)} + 3g\gamma D \right) \partial^\Lambda W + g\gamma W u^\Lambda = \\ &\qquad 9g^2 \left(v_{(\Sigma)} - v_{(\Lambda)} \right) \partial^\Sigma W \partial_\Sigma \partial^\Lambda W + 3g\gamma \left(\partial^\Sigma W \partial_\Sigma u^\Lambda - u^\Sigma \partial_\Sigma \partial^\Lambda W \right), \end{aligned} \tag{3.11}$$

This expression will be discussed in section 3.2, and will be explicitly solved for the simple models studied in section 4.

3.1 Domain wall and classification

In this section we discuss a point that is in some sense *tangential* to the main stream of the paper. We would like to shed some light on the relation between the DW solutions (and their deformations) and the classification methods developed in [53], and successfully applied to supergravity theories with 8 supercharges in [54–61, 60, 62, 63, 29, 64, 30]. In particular we want to understand within the framework of the classification, in which class the solutions we are studying fall in.[11] Although some facts and observations we report apply to diverse dimensions, we focus our discussion (as in the rest of the work) on the 5d supergravity.

Let us emphasize however that DW solutions in 5d gauged supergravity are only partially cover by the classification method. Indeed no classification in gauged supergravity with hypermultiplet couplings currently exists.

Moreover is intrinsically difficult to identify the DW and all the solutions coming from the gauging, i.e. that exist only in gauged supergravity (in the ungauged limit, $g \to 0$ reduces the vacuum). This occurs because the classification method is essentially based on the ungauged theory. Indeed, the starting point of any classification is to assume the existence of a covariantly constant spinor ϵ. This can be divided into two classes that are time-like or light-like. Such division implies the adoption of BPS solutions of ungauged supergravity as a preferred base. To see this let us recall that a solution is said time-like or light-like if the Killing vector V^μ constructed by the covariantly constant spinor ϵ,

$$V^\mu \equiv 1/2 \bar{\epsilon}^i \gamma^\mu \epsilon_i, \tag{3.12}$$

enjoys the former or the latter properties, respectively. Following [53], the modulus of V^μ can be related via Fierz identities to the scalar quantities $f \equiv 2i\bar{\epsilon}^i \epsilon_i$. A crucial consequence

[11] While this paper was being written, [24] and [25] appear. It contains some overlap with the discussion in this section.

of ϵ being covariantly constant is that V^μ turns out to be Killing. Together with the Fierz identity

$$V^a \gamma_a \epsilon_i = i f \epsilon_i, \tag{3.13}$$

it implies the existence of preferred frame, in which a projector is associated to each BPS solution:

$$\gamma_0 \epsilon = i\epsilon, \quad \text{if the (spinor, and by extension the) solution is time-like;} \tag{3.14}$$

$$\gamma_- \epsilon = 0, \quad \text{if the solution is light-like.} \tag{3.15}$$

The remarkable result [53] is that these are the only projectors possible in the ungauged supergravity[12] (as a consequence the BPS solutions are or one half or maximally supersymmetric). In this sense the classification method labels the configurations by their origins in the ungauged theory. This obviously is not all the story: the different solutions in the two classes are identified by the allowed *Base spaces*.

It worth stressing that the projector (2.25) associated to the domain wall can never be reduced to (3.14) or (3.15). Indeed, in the ungauged theory limit $g \to 0$ the algebraic condition (2.25) disappears and the domain wall reduces to maximally supersymmetric Minkowski vacuum. In the classification contest the additional projector arises checking (the assumption of) the existence of covariantly constant spinor. Indeed in the minimal gauged supergravity [54] and in the gauged supergravity with vector multiplets coupled [62] the solutions are generically 1/4 BPS. From this perspective, the BPS solutions of gauged supergravity are seen as deformations of the BPS configurations of the ungauged gravity. Such a deformation is the result of the partial supersymmetry breaking introduced by the gauging.

However, this point of view makes it difficult to characterized solutions like domain walls, which are interesting in its own and, as we remarked, are exclusively a product of the gauging. It should be noted that domain walls were not recognized in the classification up to now.

This gap can easily be filled by using the "identification" between the DW and the projector (2.25). Indeed (2.25) is only compatible with the null projection (3.15) obtained in section 2.2. Assuming instead (3.14), the anti-commuting algebra of γ-matrices is not realized on ϵ. For the same reason a projector of the form $\gamma_1 \epsilon_i = \theta^r (\sigma_r)_i{}^j \epsilon_j$ ($\theta^r \theta_r = 1$) is not compatible with either (3.14) or (3.15). This means that the coordinate transversal to the wall can not be "mixed" with time.

From these simple observations we learn that the DW can only belong to the class of light-like BPS solutions. At the same time this implies something stronger: given a domain wall solutions the only supersymmetry preserving (uncharged)*deformations admitted are null* (the ones we consider in this work) or/and have their origin in the coupling with hypermultiplets.

[12] The full supersymmetry preserving solutions fit in the above classification but are characterized by the existence of another covariantly constant spinor η satisfying the complementary projector, respectively $\gamma_0 \eta = -\eta$ and $\gamma_\pm \eta = 0$. Moreover, these are the unique configurations belonging to the both classes.

This statement reflects a peculiar point of view with respect the classification, in which the contribution of the ungauged theory are seen as perturbation of the gauged solution. This is more useful when we are interested in the properties of the latter.

Let us conclude by observing that the question over the existence of other deformations than the ones studied in this paper seems to be strictly related to the existence of 1/8 BPS solutions.

3.2 Analyzing the deformation

In section 3 we derived the BPS equation characterizing the NDDW. These equations will now be analyzed in order to understand the "physics" behind them and construct explicit solutions (see section 4). We began by reminding the reader that a NDDW can be interpret in two ways. Indeed, as the name indicates can be seen as a supersymmetric null deformation of a DW or as gauging deformations of an uncharged half BPS plane wave configuration of the ungauged supergravity theory. These "mother" classes of solutions can be derived considering the projector (2.25) and (3.1) separately. Their BPS equations are obtained from the generic case by taking the limit $u^\Lambda, v_{(\Lambda)} \to 0$ and $g \to 0$, respectively.[13] In the latter case, the kinetic term of the scalars is simple given by u^Λ.

This fact points out the "physical" difference between the null deformation controlled by u^Λ and $v_{(\Lambda)}$. We will refer to these as u-deformations and v-deformations, respectively.

The u-deformations are ungauged deformations, in the sense that u^Λ identifies the scalar profile and (up to some freedom in the function H, see section 4) the metric of the (plane wave) solution in $g \to 0$ limit. The v-deformations are instead a product of the gauging, and are the unique ones related to the potential. It follows from eq. (3.6), that the potential can be *time-dependent* (via x^+) only in presence of v-deformations. This makes it very appealing to construct these kind of solutions.

However, it seems very hard to obtain explicit solutions in the most general set-up of section 3, at least analytically. The major difficulty to overcome is the integrability condition of the scalars (3.11).

Indeed the recipe for constructing a solution consists of:

1. assigning the matter sector and the *gauging*, in practice giving a prepotential W;
2. obtaining from the SIC (3.11) admissible $v_{(\Lambda)}$ and u^Λ as function of the moduli;
3. integrating the scalar BPS equations (3.10) and determining β, $v_{(\Lambda)}$ and u^Λ as functions of spacetime (x^+ and r);
4. deriving H from (3.9).

For the first step, we note that the orthogonality between u^Λ, the Killing vector K^X and the SU(2)-covariant derivative of prepotential $D^\Lambda P^s$ requires (for $u^\Lambda \neq 0$) $n_V, n_H \neq 1$.

[13]It is worth to note that no (plane wave) solutions associated to (3.1) and supported by a non trivial potential ($W \neq$ constant) exist. This can be easily check by computing the equation of motion for the scalar (B.7) for the x^+ direction.

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The second step is certainly the most delicate. The SIC can be interpreted as an implicit definition of $v_{(\Lambda)}$ and u^Λ that otherwise do not have an well-known geometric origin as W.

The difficulties of points 3 and 4 are of technical nature, and at worst can be faced using numerical methods.

The plan above will be applied in the next section, considering u-deformation only. In this special case, more can be said on the solution. First of all, (3.6) means that $W = \frac{\gamma}{g}\frac{\dot\beta}{\beta^2}$, $\gamma = \pm 1$, is a function of r only. Moreover (3.6) tells us that $\frac{\beta'}{\beta^2} = D(x^+)$ or, in other words, that the warp-factor decomposes as follows

$$\beta^{-1} = f(r) + g(x^+). \tag{3.16}$$

Before concluding let us add a comment on the SIC (3.11). Due to the orthogonality between u^Λ and $\partial^\Lambda W$ it actually corresponds to two distinct equations. That in the u^Λ direction can be interpret as the definition of u^Λ (and $v_{(\Lambda)}$), or in other words is the consistency condition between the gauging and the ungauged solutions. That in the $\partial^\Lambda W$ direction determines D, i.e. the dependence on *time* (x^+) of the warp-factor β. Taking a constructive point of view, the first equation determines whether, for each of the possible direction orthogonal to $\partial^\Lambda W$, is possible to adjust the modulus of u^Λ in order to find a solution. In the examples we present in section 4 this occurs. Furthermore it turns out that $u^\Lambda u_\Lambda$ is not completely determined by the SIC.

4. Explicit solutions

In this section we present the explicit realization of a NDDW for the simplest models we can consider. For this purpose we restrict ourselves to u-deformation.

Indeed, as discussed above, we need *at least* $n_V \geq 2$ or/and $n_H \geq 2$. For example it is not possible to realize the orthogonality of $\partial^\Lambda W$ and u^Λ in a trivial way, i.e. taking one lying in the Hypergeometry and other in the Very Special geometry. Indeed, the integrability condition of the scalars (3.11), would force the solution to reduces to the plane way of the ungauged theory or, alternatively to a flat domain wall.[14] In that follows, we will focus over the cases: a) $(n_V, n_H) = (0, 2)$; b) $(n_V, n_H) = (2, 0)$. In particular we consider the group manifolds $\frac{\mathrm{Sp}(2,1)}{\mathrm{Sp}(2)\times\mathrm{Sp}(1)}$ and $\frac{\mathrm{SO}(2,1)}{\mathrm{SO}(2)}$.

The solutions we obtain are peculiar because the warp-factor β turns out to be a function of r only, remaining untouched by the deformation. This feature depends only on the special gauging chosen in order to guarantee the existence of analytic solutions. Why this happens is clarified in the appendix D by means of the adopted coordinates [28].

[14] This result confirms, in accordance with the expectation, that the ungauged matter sector (the one where $\partial W = 0$) decouples from the gauged matter sector and do not contribute to the solution. In other words given a domain wall supported by n_V vector multiplets and n_H hypermultiplets, an arbitrary number of constant matter multiplets can always be added.

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4.1 A $n_H = 2$ solution: the $\frac{\mathrm{Sp}(2,1)}{\mathrm{Sp}(2)\times\mathrm{Sp}(1)}$ model.

Details on the geometry and coset parametrization of the coset space $\frac{\mathrm{Sp}(2,1)}{\mathrm{Sp}(2)\times\mathrm{Sp}(1)}$ are given in the appendix C. The space is characterized by the following metric

$$\mathrm{d}s^2 = (\mathrm{d}h)^2 + (B^1)^2 + (B^2)^2 + (B^3)^2 + 2\mathrm{e}^{-h}\left[(\mathrm{d}e^0)^2 + (\mathrm{d}e^1)^2 + (\mathrm{d}e^2)^2 + (\mathrm{d}e^3)^2\right]. \quad (4.1)$$

In order to get a simple configuration, we consider as isometry to be gauged a translation. Because the metric (4.1) is cyclic in the b^r, we take

$$K = \partial_{b^1}. \quad (4.2)$$

In order to compute the prepotential P^r, we follow the same strategy as in [65]. Indeed, for practical purposes, is convenient to use another definition of P^r different than (2.7). A Killing vector preserves the connection ω^r and Kähler two forms J^r ($\frac{1}{2}\nu J^r \equiv R^r$, with $\nu = -1$ in our paper) only modulo an SU(2) rotation. Denoting by $\mathcal{L}_\Lambda$ a Lie derivative with respect to k_Λ, we have

$$\mathcal{L}_\Lambda\omega^s = -\tfrac{1}{2}\nabla r^s_\Lambda, \quad \mathcal{L}_\Lambda J^r = \varepsilon^{rst} r^s_\Lambda J^t, \quad (4.3)$$

where r^s_Λ is known as an SU(2) *compensator*. The SU(2)-bundle of a quaternionic manifold is non-trivial and therefore it is impossible to get rid of the compensator r^s_Λ by a redefinition of the SU(2) connections.[15] The moment map can be expressed in terms of the triplet of connections ω^s and the compensator r^s_Λ in the following way [66]:

$$\mathcal{P}^s_\Lambda = \tfrac{1}{2} r^s_\Lambda + \iota_\Lambda \omega^s. \quad (4.4)$$

For this Killing vector the compensator turns up to be zero, and the moment map P^r, following (4.4), is

$$P^r = \iota_K w^r = -\frac{1}{2} e^{-h}\delta^{1r}. \quad (4.5)$$

Accordingly with the BPS condition, we can choose as u any vector field in $\frac{\mathrm{Sp}(2,1)}{\mathrm{Sp}(2)\times\mathrm{Sp}(1)}$ orthogonal to K and $\iota_K J^r$, for example:

$$u = f\left(-\partial_{e^0} + e^r \partial_{b^r}\right). \quad (4.6)$$

The f at this stage is arbitrary function of the scalar manifold, but the integrability condition of the scalars will fix its dependence on the "running" ones (the others are irrelevant for determining the final solution). Considering only u-deformations, the SIC (3.11) becomes

$$3D\partial^X W + W u^X = 3\left(\partial^Y W \partial_Y u^X - u^Y \partial_Y \partial^X W\right). \quad (4.7)$$

As the superpotential W is a function of the Cartan coordinate h (with respect to which the metric (4.1) is by definition diagonal) *only*, the above equation implies

$$\begin{aligned} D &= 0 = \beta' \Longrightarrow \beta = \beta(r), \\ \partial_h \ln f &= -\frac{1}{3} \Longrightarrow f = \mathcal{C}F(q)e^{-h/3}, \qquad \partial_h F(q) = 0. \end{aligned} \quad (4.8)$$

[15] This is in contrast with $\mathcal{N} = 2$ rigid supersymmetry, since hyper-Kähler manifolds have a trivial SU(2) bundle, and therefore no compensator.

The constant $\mathcal{C}$ has been introduced for convenience. Without loosing of generality we can restrict $F(q)$ to be a function of e^0 and b^r only. As we will see, F takes the role of *generating function* of the solution. Indeed, eq. (3.10) gives

$$e^{0\prime} = -\beta f = -F, \tag{4.9}$$
$$b^{r\prime} = \beta f = F. \tag{4.10}$$

The above equations imply

$$b^r = -e^r e^0 + C^r, \qquad C^r = \text{constant},$$

and

$$\mathrm{d}x^+ = -\frac{\mathrm{d}e^0}{F(e^0, -e^r e^0 + C^r)}, \tag{4.11}$$

which can always be integrated and inverted *piecewise* for a smooth $F(e^0, b^r)$.[16]

Let us remark that the feature $\partial_+\beta = 0$ is not generic but a consequence of the simple model we have chosen. This property allows us to integrate immediately eq. (3.5):

$$h = \frac{3}{2}\ln\left[\frac{2g\gamma}{\sqrt{6}\mathcal{C}}(r - r_0)\right], \tag{4.12}$$
$$\beta = \frac{1}{\mathcal{C}}e^{h/3} = \frac{1}{\mathcal{C}}\left[\frac{2g\gamma}{\sqrt{6}\mathcal{C}}(r - r_0)\right]^{1/2}. \tag{4.13}$$

Before discussing the x^+-dependence of the solution, let us remark that at $r = r_0$ the solution has a singularity. We may take $r_0 = 0$ and chose γ in order to have the solution defined for $r > 0$. The singularity exists even when the deformation is absent. Indeed the superpotential

$$W = \frac{1}{\sqrt{6}}\left[\frac{2g\gamma}{\sqrt{6}\mathcal{C}}r\right]^{-3/2},$$

that is related to the curvature by the BPS equations, explodes for $r = 0$. This is not surprising because this happens for all DWs obtained by the gauging of a translation and for this specific model the radial dependence is unaffected by the deformation.[17]

As the warp-factor is independent of the deformation, u and the dependence on x^+ enter the metric only through the function H, describing the "wave". As explained in the previous sections, H is determined by (3.9) using the decomposition in (3.8):

$$\frac{3}{2}\frac{X}{r} + \dot{X} + Y = Br^{-3/2}, \tag{4.14}$$

where $X \equiv \dot{\tilde{H}}$, $Y \equiv \sum_i \partial_i^2 \tilde{H}$ and $B \equiv -4\left[\frac{2g\gamma}{\sqrt{6}\mathcal{C}}\right]^{-3/2} F^2$, $B < 0$. This equation can be easily integrated in r under the assumption (not required by supersymmetry) $\partial_i \dot{\tilde{H}} = 0$,

[16] However, for a generic F the solution will develop a singularity of similar kind as in [1, 2].

[17] For this specific gauging more can be said about the stringy origin of the solution. In Calabi-Yau compactification the Cartan modulus is associated to the Volume V of the compact space (to be precise $V \propto e^h$) [26]. In this specific case the singularity occurs when the CY shrinks to zero and the supergravity approximation is breaking down.

that implies $Y = Y(x^+)$. We get

$$H = -2\rho r^{-1/2} + 2Br^{1/2} - \frac{1}{5}Yr^2 + \sigma + H_- x^- , \tag{4.15}$$

with ρ and H_- generic functions of x^+, $\rho = \rho(x^+)$, $H_- = H_-(x^+)$, and $\sigma = \sigma(x^+, x^i)$ such that $\sum_i \partial_i^2 \sigma = Y$.

Let us observe that H, measuring the light-like deformation of the metric, is only partially controlled by the shape of F (via B), measuring the deformation of the scalar sector. Indeed, the presence of less supersymmetry preservation with respect to the DW allows more freedom and, in contrast with the DW case, different metrics can correspond to a single scalar profile.

Taking $F = e^0$ and fixing all the integration constant (and functions) to a convenient value, a simple solution is

$$\mathrm{d}s^2 = \frac{2}{3} gr \left(-2\mathrm{d}x^+\mathrm{d}x^- - 8g\sqrt{r}e^{-2x^+}(\mathrm{d}x^+)^2 + \mathrm{d}r^2 + (\mathrm{d}x^i)^2\right), \tag{4.16}$$

$$h = \frac{3}{2}\ln[gr], \tag{4.17}$$

$$e^0 = e^{-x^+}, \tag{4.18}$$

with the other scalars identically zero. This solution exhibits a light-like singularity for $x^+ \to -\infty$.

4.2 A $n_V = 2$ solution: the $\frac{\mathrm{SO}(2,1)}{\mathrm{SO}(2)}$ model.

We now consider the moduli space $\mathcal{M} = \frac{SO(1,n_V)}{SO(n_V)}$, $n_V > 1$. We will use the parametrization in [67]. We can then take the following polynomial

$$N(h) = \frac{3}{2}\sqrt{\frac{3}{2}}\left(\sqrt{2}h^0(h^1)^2 - h^1\left[(h^2)^2 + \ldots + (h^{n_V})^2\right]\right). \tag{4.19}$$

This means that the non-vanishing components of the tensor C_{IJK} are

$$C_{011} = \frac{\sqrt{3}}{2}, \quad C_{1ab} = -\frac{\sqrt{6}}{4}\delta_{ab}\, , \; a,b = 2,\ldots,n_V\, . \tag{4.20}$$

The constraint $N = 1$ can be solved by

$$h^0 = \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}(\varphi^1)^2} + \frac{1}{\sqrt{2}}\varphi^1\left[(\varphi^2)^2 + \ldots + (\varphi^{n_V})^2\right]\right), \tag{4.21}$$

$$h^1 = \sqrt{\frac{2}{3}}\varphi^1, \qquad h^a = \sqrt{\frac{2}{3}}\varphi^1\varphi^a. \tag{4.22}$$

Applying the Very Special geometry identities (2.3), the metric g_{xy} results diagonal in this parametrization,

$$g_{xy} = Diag\left(\frac{1}{(\phi^1)^2}, \frac{(\phi^1)^3}{3}, \ldots, \frac{(\phi^1)^3}{3}\right). \tag{4.23}$$

Here we are interested in performing a U(1) gauging. The constraint (2.9) implies for the constant P_I^r, $\vec{P}_I \times \vec{P}_J = 0$, therefore the prepotential is $P^r = P_I^r h^I$ with $P_I^r = V_I Q^r$. It follows $W = \sqrt{\frac{2}{3}} V_I h^I$. In order to get an analytic solution we choose $V_I = V\delta_I^1$. Explicitly

$$W = \frac{2}{3} V \phi^1. \tag{4.24}$$

According to the orthogonality condition we can take

$$u^x = f \delta_2^x. \tag{4.25}$$

Due to the SO(n_V) symmetry of the moduli space, u^x may always be cast in this form. This means that, for the special gauging (4.24) we can restrict without loss of generality to $n_V = 2$. As in the previous section we start by analyzing the integrability conditions for the scalars. The equation in the direction 1 gives

$$3D\partial^1 W = -3f\partial_2\partial^1 W = 0, \qquad \Rightarrow \qquad D = 0, \tag{4.26}$$

while the equation along the second component determines f

$$Wf = 3\partial^1 W \partial_1 f, \qquad \Rightarrow \qquad \partial_1 \ln f = \frac{1}{3\phi^1}. \tag{4.27}$$

Again (4.26) entails $\beta = \beta(\phi^1) = \beta(r)$ while from (4.27) follows

$$f = F(\phi^2)\beta^{-1} = (\mathcal{C})^{-1} F(\phi^2) (\phi^1)^{1/3}.$$

As in the hypermultiplet example F is completely arbitrary.

The profile of $\phi^1(r)$ can be easily determined integrating (3.5):

$$\phi^1 = \left[\frac{4}{3} g\gamma \mathcal{C} V r\right]^{-3/2}, \qquad \beta = \mathcal{C}\left[\frac{4}{3} g\gamma \mathcal{C} V r\right]^{1/2}. \tag{4.28}$$

Accidentally the solution turns out to be practically identical to one obtained in the previous section.

5. Discussion

In this paper we analyzed null deformations of flat domain wall solutions (NDDW) in gauged supergravity. In our study we used an approach mainly based on the choice of an ansatz explicitly showing, however, that we covered all the solutions of the class we were interested in. In this respect, we reviewed and further investigated the relation between the projector (2.25) and flat domain wall solutions (DW) (for related discussions see e.g. [26, 68, 69]). This allowed us to identify the DW solutions as light-like in the classification framework, and, more important, it allowed us to prove that *all the possible deformations of the DW have origin in the hypermultiplet sector* or/and *are null.*

We showed that the null deformations can have a "gauging" (v-deformation) or a "non gauging" (u-deformation) nature. This conceptual difference has practical consequences: only the presence of a v-deformation can give rise to a time-dependent (super)potential.

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As the superpotential W controls the dependence of the scalars (and, via backreaction, of the metric) on r, u^Λ and $v_{(\Lambda)}$ determine the lightcone time dependence. However, in comparison to W they do not have an intrinsic geometrical origin on the moduli space. $v_{(\Lambda)}$ and u^Λ (or better $u^\Lambda u_\Lambda$) acquire a well-defined meaning once they satisfy the integrability conditions of the scalars (SIC) as spacetime functions, $\varphi^\Lambda = \varphi^\Lambda(r, x^+)$.

The SIC play a crucial role in constructing solutions. We showed how they can be solved, and two analytical solutions supported by scalars in the hypermultiplet and in the vector sector respectively were found.

Our study also provided insights that seems to apply to generic BPS solutions in gauged supergravity [52]. We note for the first time that the compatibility of gauging imposes restrictions on the number of matter multiplets, even at the level of an abelian gauged group. Indeed if we consider u-deformation, the resulting solution can be equivalently considered as the outcome of the soft supersymmetry breaking produced by the gauging on a (null) background of the ungauged theory (identified by u^Λ). The resulting condition for preserving supersymmetry, $u^\Lambda D_\Lambda \vec{P} = u^X K_X = 0$, forces the number of matter multiplets to be different by one, $n_H, n_V \neq 1$. While for vector multiplets (where there is one scalar in each multiplet) such condition is meeting the naive expectation that for each "active" spacetime direction there is *at least* one scalar flowing (an expectation that can be made rigorous using the adapted coordinate of appendix D), this is far less obvious in the hypermultiplet case (where there are four scalars in each multiplet) and completely unexpected when both kinds are present. The above consideration reinforces the idea that it is more "natural" to regard the scalars of a hypermultiplet as a unique quaternionic scalar.

At the same time, this is an indication that an extension of the Fake supergravity formalism may be possible, at least for u-deformed DW. Roughly speaking, one expects that, in analogy with the DW, the supergravity can be effectively described by two scalars, encoding respectively x^+ and the r dependence. The second scalar should mimic only the scalars (the multiplets) involved in the gauging. Such a splitting should also appear in the fake BPS conditions. In support of this picture, we find that the democratic treatment for the scalars, introduced in [26] and extended for curved domain walls in [28], perfectly works also for NDDW.

Our results raise interesting questions that have only been considered briefly.

First of all, it would be very appealing to explore the holographic meaning of NDDW. Assuming the validity of gauged/gravity correspondence (at least when the gravity background is asymptotically AdS), one would expects that, being the deformation of AdS_5 associated to a DW and null deformation compatible at the supergravity level, the same should happen for the corresponding deformations of $\mathcal{N} = 4$ SYM. Would be very interesting to check this explicitly at the gauge theory level.

The ultimate question that naturally arises is whether the flow in (the analogue of) the radial coordinate still describes the RG flow in the dual field theory. In order to address this problem, one should take the ($\mathcal{N} = 2$ embedding of the) kinks that have dual known flow and construct their null deformation. Constructing such solutions explicitly is certainly not an easy task.

More concretely, let us consider the FGPW flow. Its $\mathcal{N} = 2$ embedding has been given in [26] in terms of one vector multiplet, one hypermultiplet with a gauging of a $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry of the scalar manifold. However, as commonly happens in presence of compact gauging with both hypers and vectors coupled, the actual solution (out of the fixed points) is known only numerically. This circumstance unfortunately makes it very difficult to solve the SIC and to construct a consistent null deformation.

On other hand, it is relatively easy to construct NDDWs based on non compact gauging, as was shown in section 4 and it should be even possible to obtain their uplifting. However, it is more difficult to find the holographic dual of such configurations because they are deformations of DWs that are not asymptotically AdS.

A possible way of circumventing such difficulties could be achieved developing a generalized Fake formalism, on the lines discussed above. Furthermore, using the technique presented in [70], one could obtained non supersymmetric but stable NDDW.

A related problem would be to consider null deformation of curved domain wall [71, 68, 69, 72, 73]. This extension can be done by simply generalizing the metric ansatz (3.2), as our calculations are valid for a generic γ, with $\gamma^2 \leq 1$. This would make possible to consider null deformation of solutions like "Janus" [74], which is conjectured to be dual to an interface CFT [75]. The stability of this ten-dimensional Type 0 solution was proven in [45] using Fake supergravity, while its embedding into $\mathcal{N} = 2$, $d = 5$ gauged supergravity has been derived in [76], following [28]. The supersymmetric Type II Janus has been recently obtained in [77] and its holographic interpretation discussed in [78].

A completely different application of the NDDW would be in the study of possible supersymmetric decay of domain walls. Very recently, in [79] it has been found that stable domain walls can asymptote to unstable anti-de Sitter vacua. The authors conjectured that these solutions decay via a time-dependent process to some near-by stable domain wall. It would be interesting to see whether a NDDW might represent a possible decay channel.

Finally, another point that deserves further attention is the existence of 1/8 BPS deformations of DWs. The very recent result of [30] suggests that solutions preserving only 1/8 of supersymmetry exist already in ungauged supergravity with hypermultiplet couplings. In contrast, without hypermultiplet coupled the supersymmetric configuration preserves *at least* two supercharges [62]. It is then reasonable to assume these deformations should exist. Characterizing such configurations is interesting on its own right and could help in the arduous task of classifying all the BPS solutions of $\mathcal{N} = 2$ gauged supergravity with hypermultiplets coupled.

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A. Metric and integrability conditions

Inspired by [1], we choose the following metric ansatz[18] (in a conformal gauge):

$$\mathrm{d}s^2 = \beta^2(x^+,r)\left(-2k^2(x^+)\mathrm{d}x^+\mathrm{d}x^- + H(x^+,x^-,x^i,r)(\mathrm{d}x^+)^2 + \mathrm{d}r^2 + (\mathrm{d}x^i)^2\right), \tag{A.1}$$

$$E^\pm = \beta\mathrm{d}x^+, \qquad E^\mp = \beta(k^2\mathrm{d}x^- - 1/2H\mathrm{d}x^+), \qquad E^i = \beta\mathrm{d}x^i, \qquad E^5 = \beta\mathrm{d}r. \tag{A.2}$$

It follows

$$w^\pm{}_\pm = \left(\frac{\beta'}{\beta^2} + \frac{2kk' + 1/2\partial_- H}{\beta k^2}\right)E^\pm, \qquad w^\pm{}_r = \frac{\dot\beta}{\beta^2}E^\pm, \tag{A.3}$$

$$w^\mp{}_i = -1/2\frac{\partial_i H}{\beta}E^\pm + \frac{\beta'}{\beta^2}E^i, \qquad w^\mp{}_r = -1/2\frac{\dot H}{\beta}E^\pm + \frac{\dot\beta}{\beta^2}E^\mp + \frac{\beta'}{\beta^2}E^5, \tag{A.4}$$

$$w^i{}_r = \frac{\dot\beta}{\beta^2}E^i, \tag{A.5}$$

where we indicate the derivative with respect to the spacetime coordinates x^+ and r with a prime and a dot, respectively: $\beta' \equiv \partial_{x^+}\beta$, $\dot\beta \equiv \partial_r\beta$. For the curvature we have

$$\Omega^{\pm\mp} = \frac{E^\pm}{\beta^2}\wedge\left[\left(\frac{\partial_-^2 H}{2k^2} - (\dot\beta/\beta)\right)^2 E^\mp + \frac{\partial_i\partial_- H}{2k^2}E^i + \left(\frac{\dot\beta'\beta - 2\dot\beta\beta'}{\beta^2} + \frac{\partial_-\dot H}{2k^2}\right)E^5\right], \tag{A.6}$$

$$\Omega^{\pm i} = -(\dot\beta/\beta^2)^2 E^\pm\wedge E^i, \qquad \Omega^{\pm r} = -\frac{\ddot\beta\beta - 2\dot\beta^2}{\beta^4}E^\pm\wedge E^5, \tag{A.7}$$

$$\begin{aligned}\Omega^{\mp i} &= 1/\beta^2 E^\pm\wedge\left\{\frac{\partial_-\partial_i H}{2k^2}E^\mp + \left[\left(\frac{\beta''\beta - 2(\beta')^2}{\beta^2} - \beta'/\beta\frac{2kk' + 1/2\partial_- H}{k^2} + \frac{\dot\beta\dot H}{2\beta}\right)\delta^i_j\right.\right.\\ &\quad \left.\left. + 1/2\partial_{ij}H\right]E^j + 1/2\partial_i\dot H E^5\right\} - \left(\frac{\dot\beta}{\beta^2}\right)^2 E^\mp\wedge E^i - \frac{\dot\beta' - 2\beta'\dot\beta}{\beta^4}E^i\wedge E^5,\end{aligned} \tag{A.8}$$

$$\begin{aligned}\Omega^{\mp r} &= 1/\beta^2 E^\pm\wedge\left[\left(\frac{\partial_-\dot H}{2k^2} + \frac{\dot\beta' - 2\dot\beta\beta'}{\beta^2}\right)E^\mp + 1/2\partial_i\dot H E^i + \left(1/2\ddot H + \frac{\dot\beta\dot H}{2\beta} + \frac{\beta''\beta - 2(\beta')^2}{\beta^2}\right.\right.\\ &\quad \left.\left. - \beta'\frac{2kk' + 1/2\partial_- H}{k^2}\right)E^5\right] - \frac{\ddot\beta\beta - (\dot\beta)^2}{\beta^4}E^\mp\wedge E^5,\end{aligned} \tag{A.9}$$

$$\Omega^{ir} = \frac{\dot\beta'\beta - 2\dot\beta\beta'}{\beta^4}E^\pm\wedge E^i - \frac{\ddot\beta\beta - (\dot\beta)^2}{\beta^4}E^i\wedge E^5, \qquad \Omega^{ij} = -\left(\frac{\dot\beta}{\beta^2}\right)^2 E^i\wedge E^j. \tag{A.10}$$

[18] With respect to [1] we define the lightcone coordinate differently, in order to have $\eta_{\pm\mp} = 1$. The main difference is that here the warp-factor β is a generic function of r and x^+. The deformed AdS metric of [1] is recovered for $\beta = 1/r$.

In that following we will use the notation $\tilde{a}$ to indicate the flat indeces different than 0, 1 ($\pm$, $\mp$).

The functions in the ansatz will be determined by the comparison with the gravitini integrality conditions (GIC) and the equations of motion for the metric. The formers are established studying the consistency of (2.20) with the projectors (2.25) and (3.1). Explicitly, from (3.4) it follows

$$
\begin{aligned}
D_a P^r &= \partial_a \varphi^\Lambda D_\Lambda P^r = \\
&= \sqrt{\frac{3}{2}} g \left[3\left(v \partial^X W \partial_X W + w \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta_a^\pm + 3\left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta_a^5 \right] \alpha^r, \\
\gamma &\equiv -\alpha^s Q^s,
\end{aligned} \tag{A.11}
$$

that implies

$$
\begin{aligned}
\Big\{ 1/2 \Omega_{cd}{}^{ab} &+ 6g^2 \left[v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right] \delta^\pm_{[c} \gamma_{d]} \gamma_5 \\
&- g^2 \left[3 \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) (\delta_c^5 + \delta_d^5) - W^2 \right] \gamma_{cd} \Big\} \epsilon_i = 0.
\end{aligned} \tag{A.12}
$$

The above equation fixes the curvature to be

$$
\Omega^{\pm\mp} = -g^2 W^2 E^\pm \wedge E^\mp - 3g^2 \left(v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right) E^\pm \wedge E^5, \tag{A.13}
$$

$$
\Omega^{\pm\tilde{a}} = g^2 \left[3 \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta^5_{\tilde{a}} - W^2 \right] E^\pm \wedge E^{\tilde{a}}, \tag{A.14}
$$

$$
\begin{aligned}
\Omega^{\mp\tilde{a}} &= \Omega_{\pm\tilde{b}}{}^{\mp\tilde{a}} E^\pm \wedge E^{\tilde{b}} - 3g^2 \left(v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta^{\tilde{a}}_5 E^\pm \wedge E^\mp \\
&\quad + g^2 \left[3 \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta^5_{\tilde{a}} - W^2 \right] E^\mp \wedge E^{\tilde{a}} \\
&\quad + 3g^2 \left(v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right) E^{\tilde{a}} \wedge E^5,
\end{aligned} \tag{A.15}
$$

$$
\begin{aligned}
\Omega^{\tilde{a}\tilde{b}} &= g^2 \left[3 \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) (\delta^5_{\tilde{a}} + \delta_{\tilde{b}}) - W^2 \right] E^{\tilde{a}} \wedge E^{\tilde{b}} \\
&\quad - 6g^2 \left(v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta^{[\tilde{a}}_{\tilde{c}} \delta^{\tilde{b}]}_5 E^\pm \wedge E^{\tilde{c}}.
\end{aligned} \tag{A.16}
$$

However, the component of the curvature $\Omega_{\pm\tilde{a}}{}^{\mp\tilde{b}}$ remains unfixed by the BPS equations, and it is determined only by the equations of motion. This is not surprising, corresponding $\Omega_{\pm\tilde{a}}{}^{\mp\tilde{b}}$ to the light-like deformation. We would like to comment that, at this stage, the integrability condition we computed applies to null-deformation of *any* domain wall, curved or flat.

As a consequence of GIC, we get the following equation for the ansatz A.1. The condition (A.13) gives

$$
\frac{1}{\beta^2} \left(\frac{\partial_-^2 H}{k^2} - \left(\frac{\dot{\beta}}{\beta} \right)^2 \right) = -g^2 W^2, \tag{A.17}
$$

together with

$$\partial_i \partial_- H = 0, \tag{A.18}$$

$$\frac{\dot\beta' - 2\dot\beta\beta'}{\beta^2} + \frac{\partial_- \dot H}{k^2} = -3g^2 \left(v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right). \tag{A.19}$$

The condition (A.14) provides

$$-\left[\frac{\ddot\beta\beta - \dot\beta^2}{\beta^4} \delta^5_{\hat a} + \left(\frac{\dot\beta}{\beta^2} \right)^2 \right] = g^2 \left[3 \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right) \delta^5_{\hat a} - W^2 \right], \tag{A.20}$$

that together with (A.17) and $\dot W = \sqrt{\frac{3}{2}} Q^s \dot\varphi^\Lambda D_\Lambda P^s = -3g\gamma\beta \left(\partial^X W \partial_X W + \frac{1}{\gamma^2} \partial^x W \partial_x W \right)$ implies

$$\frac{\dot\beta}{\beta^2} = g\gamma W, \qquad \gamma^2 = 1, \tag{A.21}$$

and

$$\partial_-^2 H = 0. \tag{A.22}$$

In comparison with the DW case, we observe how the radial dependence of the warp-factor is still controlled by the superpotential, with the difference that now W can be also a function of x^+. As well, the relation $\gamma^2 = 1$ indicating the "flatness" of the wall is maintained: actually this is an input we put in (A.1), focusing as announced on deformation of the (flat) DW metric (2.38). The condition (A.15) brings to

$$\frac{\dot\beta' - 2\dot\beta\beta'}{\beta^2} = -3g^2 \left(v\, \partial^X W \partial_X W + w\, \frac{1}{\gamma^2} \partial^x W \partial_x W \right), \tag{A.23}$$

$$\partial_- \dot H = 0. \tag{A.24}$$

The condition (A.16) does not furnish any new relation. The independent equations for the ansatz are summarized in the main text, eq. (3.5), (3.7). The additional equation necessary to determine H in terms of the geometric quantities W, v, w and u^Λ will come from the equations of motion, eq. (B.6).

The last integrability condition to be considered comes from the scalar fields. Indeed, being now functions of r and x^+, is necessary to check that $\partial_{[r}\partial_{x^+]}\varphi^\Lambda = 0$. The explicit expression is given in section 3, eq. (3.11).

B. Equations of motion

The equations of motion of the lagrangian (2.11) (taking in account also the terms containing the gauge field that are zero for the configurations we study) for the metric, the gauge field and the scalars are, respectively

$$-R_{\mu\nu} + a_{IJ} F^I_{\mu a} F^{Ja}_\nu + g_{XY} D_\mu q^X D_\nu q^Y + g_{xy} D_\mu \phi^x D_\nu \phi^y - \frac{1}{6}|F|^2 g_{\mu\nu} + \frac{2}{3} g^2 \mathcal{V} g_{\mu\nu} = 0, \tag{B.1}$$

$$\nabla_a (a_{IK} F^{\mathrm{Kae}}) + \frac{1}{2\sqrt6} C_{IJK} \epsilon^{abcde} F^J_{ab} F^K_{cd} - g K^X_I D^e q^Y g_{XY} = 0, \tag{B.2}$$

$$\hat{D}_\mu(D^\mu q^W) + gA^{\mu I} D_\mu K_I^W = g^2 g^{WX}\partial_X \mathcal{V}, \tag{B.3}$$

$$\hat{D}_\mu(D^\mu \phi^x) + gA^{\mu I} D_\mu K_I^x = g^2 g^{xy}\partial_y \mathcal{V} + \frac{1}{4} g^{xy}\partial_y a_{IJ} F^I_{\mu\nu} F^{J\mu\nu}, \tag{B.4}$$

where $\hat{D}$ is a totally covariant derivative, ie with respect to all the indices, explicitly

$$\hat{D}_\mu D^\mu \varphi^\Lambda = \nabla_\mu D^\mu \varphi^\Lambda + \Gamma^\Lambda_{\Sigma\Theta} D_\mu \varphi^\Sigma D^\mu \varphi^\Theta.$$

Specializing them to our uncharged configurations we get for the metric (we use the unifying notation for the scalars)

$$R_{ab} = g_{\Lambda\Sigma}\partial_a \varphi^\Lambda \partial_b \varphi^\Sigma + \frac{2}{3} g^2 \mathcal{V}\eta_{ab}. \tag{B.5}$$

This identity can be easily checked for the component of the Ricci tensor following by the integrability condition (A.12) and the correspondent BPS values of the kinetic term of the scalars (3.10) and of the potential (2.19). Such result can be obtained applying the general result of [32]. The $(\pm,\pm)$- component gives instead a new equation:

$$R_{\pm\pm} = -9g^2 \left(v\, \partial^X W \partial_X W + w\, \partial^x W \partial_x W \right) - u^\Lambda u_\Lambda. \tag{B.6}$$

Making the comparison with the metric ansatz (A.1) one finds the constraint (3.9).

Although we are considering uncharged configuration (B.2) is not trivial. Indeed it entails $K^X \partial_\mu q^Y g_{XY} = 0$. The main consequence of such condition is that the Hyperini equation becomes of the same form of the gaugini equation. This fact may be seen as the deepest reason why the democratic treatment of the scalars applies in the contest of DW solutions.

Regarding the e.o.m for the scalars, we observe that it reduces to the equation for the DW. Explicitly, we have

$$\nabla_\mu(\partial^\mu \varphi^\Lambda) + \Gamma^\Lambda_{\Omega\Sigma}\partial_\mu \varphi^\Omega \partial^\mu \varphi^\Sigma = g^2 \partial^\Lambda \mathcal{V}, \tag{B.7}$$

where the first term on the l.h.s. can be written as $\nabla_\mu(\partial^\mu\varphi^\Lambda) = \partial_\mu\partial^\mu\varphi^\Lambda + (\partial_\mu \ln\sqrt{-g})\partial^\mu\varphi^\Lambda$. It is immediate to verify that only the term with $\mu = r$ survives because NDDWs are cyclic in x^- and $g^{\mu\nu}$ is off-diagonal in x^+. Hence, (B.7) reduces to scalar e.o.m. of the DW and the same manipulations hold, as the relation between $\partial_r\varphi^\Lambda$, β and W is unchanged in the deformed case.

C. Parametrization of the two-dimensional projective quaternionic space

We shall consider the quaternionic-Kähler manifold of quaternionic dimension 2:

$$\frac{\mathrm{Sp}(2,1)}{\mathrm{Sp}(2)\times\mathrm{Sp}(1)} \simeq \frac{\mathrm{USp}(4,2)}{\mathrm{USp}(4)\times\mathrm{USp}(2)}. \tag{C.1}$$

The algebra of the isometry group, $\mathfrak{sp}(2,1)$ can be defined as the set of matrices over the quaternions $\mathbb{H}$ that preserve a metric of signature $(+,+,-)$. We take this metric in the form

$$\mu = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}, \tag{C.2}$$

where each entry is a quaternion, or 2×2 complex matrix. The elements M of $\mathfrak{sp}(2,1)$ are those 3×3 matrices with entries in $\mathbb{H}$ that satisfy

$$\mu M^\dagger \mu = -M. \tag{C.3}$$

The general form of an element of $\mathfrak{sp}(2,1)$ is then

$$M = \begin{pmatrix} a & \frac{1}{2}(\bar{e}+\bar{f}) & -\frac{1}{2}(\vec{b}+\vec{c}) \\ \frac{1}{2}(e-f) & \vec{p} & -\frac{1}{2}(e+f) \\ \frac{1}{2}(\vec{b}-\vec{c}) & \frac{1}{2}(\bar{f}-\bar{e}) & -\bar{a} \end{pmatrix}, \tag{C.4}$$

where $a = a_0 + \vec{a}$, $e = e_0 + \vec{e}$ and $f = f_0 + \vec{f}$ are generic quaternions and $\vec{c}$, $\vec{b}$ and $\vec{p}$ are pure anti-Hermitian quaternions (with vanishing Hermitian part).[19]

The Lie algebra of $\mathfrak{sp}(2,1)$ can be split into a compact (anti-Hermitian) and non-compact (Hermitian) part :

$$M_H = \begin{pmatrix} \vec{a} & \frac{1}{2}\bar{f} & -\frac{1}{2}\vec{c} \\ -\frac{1}{2}f & \vec{p} & -\frac{1}{2}f \\ -\frac{1}{2}\vec{c} & \frac{1}{2}\bar{f} & \vec{a} \end{pmatrix}, \quad M_{G/H} = \begin{pmatrix} a_0\mathbb{1} & \frac{1}{2}\bar{e} & -\frac{1}{2}\vec{b} \\ \frac{1}{2}e & 0 & -\frac{1}{2}e \\ \frac{1}{2}\vec{b} & -\frac{1}{2}\bar{e} & -a_0\mathbb{1} \end{pmatrix}. \tag{C.5}$$

The H part of the generator can be decomposed into its subalgebras:[20]

$$M_{\mathfrak{su}(2)} = \begin{pmatrix} \vec{u} & 0 & -\vec{u} \\ 0 & 0 & 0 \\ -\vec{u} & 0 & \vec{u} \end{pmatrix}, \quad M_{\mathfrak{sp}(2)} = \begin{pmatrix} \vec{v} & \frac{1}{2}\bar{f} & \vec{v} \\ -\frac{1}{2}f & \vec{p} & -\frac{1}{2}f \\ \vec{v} & \frac{1}{2}\bar{f} & \vec{v} \end{pmatrix}. \tag{C.6}$$

$M_{\mathfrak{sp}(1)}$ commutes with $M_{\mathfrak{sp}(2)}$ and the latter contains two commuting $\mathfrak{su}(2)$ parameterized by $\vec{p}$ and $\vec{v}$:

$$M_{\mathfrak{su}(2)\oplus\mathfrak{su}(2)\subset\mathfrak{sp}(2)} = \begin{pmatrix} \vec{v} & 0 & \vec{v} \\ 0 & \vec{p} & 0 \\ \vec{v} & 0 & \vec{v} \end{pmatrix}. \tag{C.7}$$

We see that the compact subalgebra of $\mathfrak{sp}(2,1)$ contains three commuting $\mathfrak{su}(2)$. $M_{\mathfrak{su}(2)} \subset \mathfrak{sp}(1)$ corresponds to the R-symmetry whereas the $\mathfrak{su}(2)_{\vec{p}} \subset \mathfrak{sp}(2)$ contains the compact U(1) for the string.

The solvable gauge of the coset manifold is obtained by adding to $M_{G/H}$ an element of M_H (with $\vec{c} = \vec{b}$, $f = e$ and $\vec{a} = \vec{p} = 0$) so that the result is an upper triangular matrix:

$$M_{\text{Solvable}} = \begin{pmatrix} a_0\mathbb{1} & \bar{e} & -\vec{b} \\ 0 & 0 & -e \\ 0 & 0 & -a_0\mathbb{1} \end{pmatrix}. \tag{C.8}$$

[19] The identification $\mathfrak{sp}(2,1) \simeq \mathfrak{usp}(4,2)$ is obtained once we take the matrices $-i\vec{\sigma}$ for the imaginary quaternions.

[20] It is related to the previous expression of M_H by taking $\vec{u} = \frac{1}{2}\vec{a} + \frac{1}{4}\vec{c}$ and $\vec{v} = \frac{1}{2}\vec{a} - \frac{1}{4}\vec{c}$.

C.1 Solvable coordinates and metric of $\frac{\mathrm{Sp}(2,1)}{\mathrm{Sp}(2)\,\mathrm{Sp}(1)}$

We parametrize the coset elements by

$$L = \mathrm{e}^{N} \cdot \mathrm{e}^{H}, \tag{C.9}$$

where

$$N = N_e + N_b = \underbrace{\begin{pmatrix} 0 & \bar{e} & 0 \\ 0 & 0 & -e \\ 0 & 0 & 0 \end{pmatrix}}_{N_e} + \underbrace{\begin{pmatrix} 0 & 0 & -\vec{b} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{N_b}, \quad H = \frac{1}{2}\begin{pmatrix} h\mathbb{1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -h\mathbb{1} \end{pmatrix}. \tag{C.10}$$

The coordinates q^X are thus the real h, the 3 real coordinates of $\vec{b}$ and the 4 real parts of the quaternion e. This leads to

$$L = \begin{pmatrix} \mathrm{e}^{\frac{1}{2}h}\mathbb{1} & \bar{e} & -\mathrm{e}^{-\frac{1}{2}h}(\vec{b} + \frac{\bar{e}e}{2}) \\ 0 & \mathbb{1} & -\mathrm{e}^{-\frac{1}{2}h} e \\ 0 & 0 & \mathrm{e}^{-\frac{1}{2}h}\mathbb{1} \end{pmatrix}. \tag{C.11}$$

This leads to the algebra element

$$L^{-1}\mathrm{d}L = \begin{pmatrix} \frac{B_0}{2} & \frac{\bar{E}}{\sqrt{2}} & -\vec{B} \\ 0 & 0 & -\frac{E}{\sqrt{2}} \\ 0 & 0 & -\frac{B_0}{2} \end{pmatrix}, \tag{C.12}$$

where

$$B = B_0\mathbb{1} + \vec{B} = \mathrm{d}h\mathbb{1} + \mathrm{e}^{-h}\left[\mathrm{d}\vec{b} - \tfrac{1}{2}(\bar{e}\mathrm{d}e - \mathrm{d}\bar{e}e)\right], \quad E = \sqrt{2}\,\mathrm{e}^{-\frac{1}{2}h}\mathrm{d}e, \tag{C.13}$$

or in real components

$$B_0 = \mathrm{d}h, \qquad B^r = \mathrm{e}^{-h}\left(\mathrm{d}b^r + e^r\mathrm{d}e^0 - e^0\mathrm{d}e^r - \varepsilon^{rst}e^s\mathrm{d}e^t\right). \tag{C.14}$$

The algebra element can be split in the coset part and the part in H. The first one is the Hermitian part:

$$(L^{-1}\mathrm{d}L)_{G/H} = \frac{1}{2}\begin{pmatrix} B_0 & \frac{\bar{E}}{\sqrt{2}} & -\vec{B} \\ \frac{E}{\sqrt{2}} & 0 & -\frac{E}{\sqrt{2}} \\ \vec{B} & -\frac{\bar{E}}{\sqrt{2}} & -B_0. \end{pmatrix}. \tag{C.15}$$

The part in H is the anti-Hermitian part, which can be split in the $\mathfrak{sp}(1)$ and $\mathfrak{sp}(2)$ part:

$$\begin{aligned}
(L^{-1}\mathrm{d}L)_H &= \frac{1}{2}\begin{pmatrix} 0 & \frac{\bar{E}}{\sqrt{2}} & -\vec{B} \\ -\frac{E}{\sqrt{2}} & 0 & -\frac{E}{\sqrt{2}} \\ -\vec{B} & \frac{\bar{E}}{\sqrt{2}} & 0 \end{pmatrix} = (L^{-1}\mathrm{d}L)_{\mathfrak{sp}(1)} + (L^{-1}\mathrm{d}L)_{\mathfrak{sp}(2)}, \\
(L^{-1}\mathrm{d}L)_{\mathfrak{sp}(1)} &= \frac{1}{4}\begin{pmatrix} \vec{B} & 0 & -\vec{B} \\ 0 & 0 & 0 \\ -\vec{B} & 0 & \vec{B} \end{pmatrix}, \\
(L^{-1}\mathrm{d}L)_{\mathfrak{sp}(2)} &= \begin{pmatrix} -\frac{1}{4}\vec{B} & \frac{\bar{E}}{\sqrt{2}} & -\frac{1}{4}\vec{B} \\ -\frac{E}{\sqrt{2}} & 0 & -\frac{E}{\sqrt{2}} \\ -\frac{1}{4}\vec{B} & \frac{\bar{E}}{\sqrt{2}} & -\frac{1}{4}\vec{B} \end{pmatrix}.
\end{aligned} \tag{C.16}$$

The metric is defined as

$$\mathrm{d}s^2 = g_{XY}\mathrm{d}q^X\mathrm{d}q^Y = \mathrm{Tr}\left[(L^{-1}\mathrm{d}L)_{G/H}\cdot(L^{-1}\mathrm{d}L)_{G/H}\right] = \tfrac{1}{2}\,\mathrm{tr}(B\bar{B}+E\bar{E}), \tag{C.17}$$

where Tr stands for a trace over the 6×6 matrix and tr for a trace over the 2×2 matrix. We will comment on the normalization of this metric below. Its value is

$$\mathrm{d}s^2 = (\mathrm{d}h)^2 + (B^1)^2 + (B^2)^2 + (B^3)^2 + 2\mathrm{e}^{-h}\left[(\mathrm{d}e^0)^2 + (\mathrm{d}e^1)^2 + (\mathrm{d}e^2)^2 + (\mathrm{d}e^3)^2\right]. \tag{C.18}$$

The vielbeins, as 1-forms and quaternions as explained above, can be taken to be

$$f^1 = \frac{1}{\sqrt{2}}B, \qquad f^2 = \frac{1}{\sqrt{2}}E. \tag{C.19}$$

These lead to (C.17) and to the hypercomplex form ($\wedge$ symbols understood)

$$\vec{J} = -\tfrac{1}{2}\left(\bar{B}\,B + \bar{E}\,E\right), \qquad \text{or} \qquad J^r = -B_0B^r - E_0E^r - \tfrac{1}{2}\varepsilon^{rst}\left(B^sB^t + E^sE^t\right). \tag{C.20}$$

Using the differentials

$$\begin{aligned} &\mathrm{d}B = -B_0B - \tfrac{1}{2}\bar{E}\,E, \qquad \mathrm{d}E = -\tfrac{1}{2}B_0E, \\ \text{or}\qquad &\mathrm{d}B^r = -B_0B^r - E_0E^r - \tfrac{1}{2}\varepsilon^{rst}E^sE^t, \end{aligned} \tag{C.21}$$

we obtain

$$\mathrm{d}J^r + 2\varepsilon^{rst}\omega^sJ^t = 0, \tag{C.22}$$

for

$$\omega^r = -\tfrac{1}{2}B^r. \tag{C.23}$$

We find then that (2.8) is satisfied for $\nu = -1$. The value that we get here for ν depends on the normalization of the metric. Multiplying the metric by an arbitrary $-\nu^{-1}$, would lead to (2.8) with this arbitrary value of ν. In the supergravity context, $\nu = -\kappa^2$, where κ is the gravitational coupling constant, which we have put equal to 1.

D. Adapted coordinates

In this section we present some insights on the SIC and the possible solutions it admits. The crucial ingredient is the adoption of adapted coordinates, associated to an existing solution. This choice allows to emphasize the physics of the solution, that is characterized by two dynamical scalars. As a result, the properties of the possible solutions can be better understood, even without constructing them explicitly. However, one has to have clear the price paid assuming a priori the existence of a solution. We will further comment on this point.

Generalizing the argument in [28], the metric of the moduli space on the two-dimensional sub-manifold identified by a solution can be cast as

$$g_{\Lambda\Sigma}|_{\mathrm{sol}} = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{12} & g_{22} & 0 \\ 0 & 0 & g_{\hat{\Lambda}\hat{\Sigma}} \end{pmatrix}, \tag{D.1}$$

where φ^1 and φ^2 represent the dynamical scalars while the others are constant. In this optic the meaning of the SIC is more clear. The different solutions for a given W coincides with the possible embedding of a two-dimensional submanifolds $\mathcal{I}_{\rm sol}$ admitting a metric of the form (D.1).

To be concrete, let us study this problem in presence of u-deformation only. In this special case, as observed in section 3.2, the r and x^+ dependence "decouple", being $W = W(r)$ and $D = D(x^+)$. This further simplifies our problem. $W = W(r)$ implies the existence of a preferred coordinate system in which $\varphi^1 = W$. This parametrization is well-defined until we are out of the critical points of superpotential, i.e. $\partial_\Lambda W \neq 0$. The clear advantage of this coordinate choice is that u^Λ lies in the direction 2 and φ^1 depends only on r, $u^1 = (\varphi^1)' = 0$. Because of this (3.11) gives

$$Dg^{11} = -u^2\partial_2(g^{11}), \tag{D.2}$$

$$Dg^{12} + \frac{1}{3}\varphi^1 u^2 = (g^{11}\partial_1 + g^{12}\partial_2)u^2 - u^2\partial_2 g^{12}. \tag{D.3}$$

These equations can be formally integrated in terms of φ^1 and φ^2 taking in account that $(\varphi^1)' = 0$ together with (D.2) implies

$$\frac{D}{u^2} = \partial_2 \ln\beta = -\partial_2 \ln g^{11}. \tag{D.4}$$

This leads to a warp-factor of the form $\beta = \frac{F(\varphi^1)}{g^{11}}$; the function F can be computed using (D.2) or equivalently (3.5). The resulting $\beta(\varphi^1, \varphi^2)$ is

$$\beta = e^{-\frac{1}{3}\left[\int\left(g^{12}\partial_2(\frac{1}{g^{11}}) - \frac{\varphi^1}{g^{11}}\right)\mathrm{d}\varphi^1\right]}. \tag{D.5}$$

Equivalently, a formal expression for u^2 can be obtained from (D.3) solving

$$(g^{11}\partial_1 + g^{12}\partial^{12})\ln u^2 = \partial_2 g^{12} - g^{12}\partial_2 \ln g^{11} + \varphi^1. \tag{D.6}$$

As in the explicit example of section 4 u^Λ is not completely determined as a function of the moduli space. In terms of the previous equations, the spacetime parametrization of the scalars is

$$\dot\varphi^1 = -3g\gamma\beta g^{11} = -3g\gamma F(\varphi_1), \tag{D.7}$$

$$\dot\varphi^2 = -3g\gamma\beta g^{12} = -3g\gamma\frac{g^{12}}{g^{11}}F(\varphi_1), \tag{D.8}$$

$$\varphi^{2\prime} = \beta u^2 = \frac{u^2}{g^{11}}F(\varphi_1). \tag{D.9}$$

The above equations deserve some comments. As we stress at the beginning they can be interpret only as a formal solution. Indeed, we start assuming that the solution exists: this implies that the coefficients of the effective two dimensional metric are non generic, in order to guarantee the existence of the solution. This can be understood considering $F(\varphi_1)$: $\partial_2 F = 0$ ends up in an integrability condition on the such coefficients. Hence the integrability requirements of SIC are just rewritten in a different way.

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Keeping in mind this caveat, the adapted coordinates are still an useful tool. For example, they make clear that we may have a solution with non trivial D even if the two dimensional metric is diagonal, i.e. $g^{12} = 0$. As (D.4) shows, the crucial condition is $\partial_2 g^{11} \neq 0$. Unfortunately, this does not occur for the examples considered in section 4.

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